



## CHAUDHARY DEVI LAL UNIVERSITY, SIRSA

(Establishment by the State Legislature Act 9 of 2003)

### DEPARTMENT OF MATHEMATICS

#### Syllabi for Entrance Examination

#### M.Sc. (Mathematics)

**Algebra. (03 Questions)** Symmetric, Skew symmetric, Hermitian and skew Hermitian matrices. Elementary Operations on matrices. Rank of a matrices. Inverse of a matrix. Linear dependence and independence of rows and columns of matrices. Row rank and column rank of a matrix. Eigenvalues, eigenvectors and the characteristic equation of a matrix. Minimal polynomial of a matrix. Cayley Hamilton theorem and its use in finding the inverse of a matrix. Applications of matrices to a system of linear (both homogeneous and non-homogeneous) equations. Theoremson consistency of a system of linear equations. Unitary and Orthogonal Matrices, Bilinear and Quadratic forms. Relations between the roots and coefficients of general polynomial equation in one variable. Solutions of polynomial equations having conditions on roots. Common roots and multiple roots. Transformation of equations. Nature of the roots of an equation Descarte's rule of signs. Solutions of cubic equations (Cardon's method). Biquadratic equations and theirsolutions.

**Calculus.(03 Questions)** Definition of the limit of a function. Basic properties of limits, Continuous functions and classification of discontinuities. Differentiability. Successive differentiation. Leibnitz theorem. Maclaurin and Taylor series expansions. Asymptotes in Cartesian coordinates, intersection of curve and its asymptotes, asymptotes in polar coordinates. Curvature, radius of curvature for Cartesian curves, parametric curves, polar curves. Newton's method. Radius of curvature for pedal curves. Tangential polar equations. Centre of curvature. Circle of curvature. Chord of curvature, evolutes. Tests for concavity and convexity. Points of inflexion. Multiplepoints. Cusps, nodes & conjugate points. Type of cusps. Tracing of curves in Cartesian, parametric and polar co-ordinates. Reduction formulae. Rectification, intrinsic equations of curve. Quardrature (area)Sectorial area. Area bounded by closed curves. Volumes and surfaces of solids of revolution. Theorems of Pappu's and Guilden.

**Solid Geometry.(03 Questions)** General equation of second degree. Tracing of conics. Tangent at any point to the conic, chord of contact, pole of line to the conic, director circle of conic. Systemof conics. Confocal conics. Polar equation of a conic, tangent and normal to the conic. Sphere:Plane section of a sphere. Sphere through a given circle. Intersection of two spheres, radical plane of two spheres. Co-oxal system of spheres. Cones. Right circular cone, enveloping cone and reciprocal cone. Cylinder: Right circular cylinder and enveloping cylinder. Central Conicoids: Equation of tangent plane. Director sphere. Normal to the conicoids. Polar plane of a point. Enveloping cone of a

coincoid. Enveloping cylinder of a coincoid. Paraboloids: Circular section, Plane sections of coincoids. Generating lines. Confocal coincoid. Reduction of second degree equations.

**Number Theory and Trigonometry. (03 Questions)** Divisibility, G.C.D. (greatest common divisors), L.C.M. (least common multiple). Primes, Fundamental Theorem of Arithmetic. Linear Congruences, Fermat's theorem. Wilson's theorem and its converse. Linear Diophantine equations in two variables. Complete residue system and reduced residue system modulo  $m$ . Euler's  $\phi$  function Euler's generalization of Fermat's theorem. Chinese Remainder Theorem. Quadratic residues. Legendre symbols. Lemma of Gauss; Gauss reciprocity law. Greatest integer function  $[x]$ . The number of divisors and the sum of divisors of a natural number  $n$  (The functions  $d(n)$  and  $\sigma(n)$ ). Mobius function and Mobius inversion formula. De Moivre's Theorem and its Applications. Expansion of trigonometrical functions. Direct circular and hyperbolic functions and their properties. Inverse circular and hyperbolic functions and their properties. Logarithm of a complex quantity. Gregory's series. Summation of Trigonometry series.

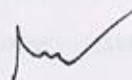
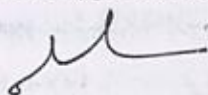
**Ordinary Differential Equations. (03 Questions)** Geometrical meaning of a differential equation. Exact differential equations, integrating factors. First order higher degree equations solvable for  $x, y, p$  Lagrange's equations, Clairaut's equations. Equation reducible to Clairaut's form. singular solutions. Orthogonal trajectories: in Cartesian coordinates and polar coordinates. Self orthogonal family of curves. Linear differential equations with constant coefficients. Homogeneous linear ordinary differential equations. Equations reducible to homogeneous linear ordinary differential equations. Linear differential equations of second order: Reduction to normal form. Transformation of the equation by changing the dependent variable/ the independent variable. Solution by operators of non-homogeneous linear differential equations. Reduction of order of a differential equation. Method of variations of parameters. Method of undetermined coefficients. Total differential equations. Condition for  $Pdx + Qdy + Rdz = 0$  to be exact. General method of solving  $Pdx + Qdy + Rdz = 0$  by taking one variable constant. Method of auxiliary equations.

**Vector Calculus. (03 Questions)** Scalar and vector product of three vectors, product of four vectors. Reciprocal vectors. Vector differentiation. Scalar Valued point functions, vector valued point functions, derivative along a curve, directional derivatives. Gradient of a scalar point function, geometrical interpretation of grad., character of gradient as a point function. Divergence and curl of vector point function, characters of Div and Curl as point function, examples. Gradient, divergence and curl of sums and product and their related vector identities. Laplacian operator. Orthogonal curvilinear coordinates Conditions for orthogonality fundamental triad of mutually orthogonal unit

vectors. Gradient, Divergence, Curl and Laplacian operators in terms of orthogonal curvilinear coordinates, Cylindrical co-ordinates and Spherical co-ordinates. Vector integration; Line integral, Surface integral, Volume integral. Theorems of Gauss, Green & Stokes and problems based on these theorems.

**Advanced Calculus. (03 Questions)** Continuity, Sequential Continuity, properties of continuous functions, Uniform continuity, chain rule of differentiability. Mean value theorems; Rolle's Theorem and Lagrange's mean value theorem and their geometrical interpretations. Taylor's Theorem with various forms of remainders, Darboux intermediate value theorem for derivatives, Indeterminate forms. Limit and continuity of real valued functions of two variables. Partial differentiation. Total Differentials; Composite functions & implicit functions. Change of variables. Homogenous functions & Euler's theorem on homogeneous functions. Taylor's theorem for functions of two variables. Differentiability of real valued functions of two variables. Schwarz and Young's theorem. Implicit function theorem. Maxima, Minima and saddle points of two variables. Lagrange's method of multipliers. Curves: Tangents, Principal normals, Binormals, Serret-Frenet formulae. Locus of the centre of curvature, Spherical curvature, Locus of centre of Spherical curvature, Involutives, evolutes, Bertrand Curves. Surfaces: Tangent planes, one parameter family of surfaces, Envelopes. ff

**Partial Differential Equations.(03 Questions)** Partial differential equations: Formation, order and degree, Linear and Non-Linear Partial differential equations of the first order: Complete solution, singular solution, General solution, Solution of Lagrange's linear equations, Charpit's general method of solution. Compatible systems of first order equations, Jacobi's method. Linear partial differential equations of second and higher orders, Linear and non-linear homogeneous and non-homogeneous equations with constant coefficients, Partial differential equation with variable coefficients reducible to equations with constant coefficients, their complimentary functions and particular Integrals, Equations reducible to linear equations with constant coefficients. Classification of linear partial differential equations of second order, Hyperbolic, parabolic and elliptic types, Reduction of second order linear partial differential equations to Canonical (Normal) forms and their solutions, Solution of linear hyperbolic equations, Monge's method for partial differential equations



of second order. Cauchy's problem for second order partial differential equations, Characteristic equations and characteristic curves of second order partial differential equation, Method of separation of variables: Solution of Laplace's equation, Wave equation (one and two dimensions), Diffusion (Heat) equation (one and two dimension) in Cartesian Co-ordinate system.

**Statics.(02 Questions)** Composition and resolution of forces. Parallel forces. Moments and Couples. Analytical conditions of equilibrium of coplanar forces. Friction. Centre of Gravity. Virtual work. Forces in three dimensions. Poinots central axis. Wrenches. Null lines and planes. Stable and unstable equilibrium.

**Sequences and Series.(03 Questions)** Boundedness of the set of real numbers; least upper bound, greatest lower bound of a set, neighborhoods, interior points, isolated points, limit points, open sets, closed set, interior of a set, closure of a set in real numbers and their properties. Bolzano-Weiestrass theorem, Open covers, Compact sets and Heine-Borel Theorem. Sequence: Real Sequences and their convergence, Theorem on limits of sequence, Bounded and monotonic sequences, Cauchy's sequence, Cauchy general principle of convergence, Subsequences, Subsequential limits. Infinite series: Convergence and divergence of Infinite Series, Comparison Tests of positive terms Infinite series, Cauchy's general principle of Convergence of series, Convergence and divergence of geometric series, Hyper Harmonic series or p-series. Infinite series: D-Alembert's ratio test, Raabe's test, Logarithmic test, de Morgan and Bertrand's test, Cauchy's Nth root test, Gauss Test, Cauchy's integral test, Cauchy's condensation test. Alternating series, Leibnitz's test, absolute and conditional convergence, Arbitrary series: abel's lemma, Abel's test, Dirichlet's test, Insertion and removal of parenthesis, re-arrangement of terms in a series, Dirichlet's theorem, Riemann's Re-arrangement theorem, Pringsheim's theorem (statement only), Multiplication of series, Cauchy product of series, (definitions and examples only) Convergence and absolute convergence of infinite products.

**Special Functions and Integral Transforms.(02 Questions)** Series solution of differential equations- Power series method, Definitions of Beta and Gamma functions. Bessel equation and its solution: Bessel functions and their properties-Convergence, recurrence, Relations and generating functions, Orthogonality of Bessel functions. Legendre and Hermite differentials equations and their solutions: Legendre and Hermite functions and their properties-Recurrence Relations and generating functions. Orthogonality of Legendre and Hermite polynomials. Rodrigues' Formula for Legendre & Hermite Polynomials, Laplace Integral Representation of Legendre polynomial. Laplace Transforms - Existence theorem for Laplace transforms, Linearity of the Laplace transforms, Shifting

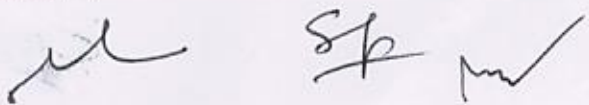
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- theorems, Laplace transforms of derivatives and integrals, Differentiation and integration of Laplace transforms, Convolution theorem, Inverse Laplace transforms, convolution theorem, Inverse Laplace transforms of derivatives and integrals, solution of ordinary differential equations using Laplace transform. Fourier transforms: Linearity property, Shifting, Modulation, Convolution Theorem, Fourier Transform of Derivatives, Relations between Fourier transform and Laplace transform, Parseval's identity for Fourier transforms, solution of differential Equations using Fourier Transforms.

**Programming in C.(02 Questions)** Programmer's model of a computer, Algorithms, Flow charts, Data types, Operators and expressions, Input / outputs functions. Decisions control structure: Decision statements, Logical and conditional statements, Implementation of Loops, Switch Statement & Case control structures. Functions, Preprocessors and Arrays. Strings: Character Data Type, Standard String handling Functions, Arithmetic Operations on Characters. Structures: Definition, using Structures, use of Structures in Arrays and Arrays in Structures. Pointers: Pointers Data type, Pointers and Arrays, Pointers and Functions.

**Real Analysis.(03 Questions)** Riemann integral, Integrability of continuous and monotonic functions, The Fundamental theorem of integral calculus. Mean value theorems of integral calculus. Improper integrals and their convergence, Comparison tests, Abel's and Dirichlet's tests, Frullani's integral, Integral as a function of a parameter. Continuity, Differentiability and integrability of an integral of a function of a parameter. Definition and examples of metric spaces, neighborhoods, limit points, interior points, open and closed sets, closure and interior, boundary points, subspace of a metric space, equivalent metrics, Cauchy sequences, completeness, Cantor's intersection theorem, Baire's category theorem, contraction Principle. Continuous functions, uniform continuity, compactness for metric spaces, sequential compactness, Bolzano-Weierstrass property, total boundedness, finite intersection property, continuity in relation with compactness, connectedness, components, continuity in relation with connectedness.

**Groups and Rings.(03 Questions)** Definition of a group with example and simple properties, groups, Subgroups and Subgroup criteria, Generation of groups, cyclic groups, Cosets, Left and right cosets, Index of a sub-group Coset decomposition, Lagrange's theorem and its consequences, Normal subgroups, Quotient groups. Homomorphism, isomorphism, automorphism and inner automorphism of a group. Automorphism of cyclic groups, Permutations groups. Even and odd permutations. Alternating groups, Cayley's theorem, Center of a group and derived group of



group. Introduction to rings, subrings, integral domains and fields, Characteristics of a ring. Ring homomorphisms, ideals (principal, prime and Maximal) and Quotient rings, Field of quotients of an integral domain. Euclidean rings, Polynomial rings, Polynomials over the rational field, The Eisenstein's criterion, Polynomial rings over commutative rings, Unique factorization domain, R unique factorization domain implies so is  $R[X_1, X_2, \dots, X_n]$

**Dynamics.(02 Questions)** Velocity and acceleration along radial, transverse, tangential and normal directions. Relative velocity and acceleration. Simple harmonic motion. Elastic strings. Mass, Momentum and Force. Newton's laws of motion. Work, Power and Energy. Definitions of Conservative forces and Impulsive forces. Motion on smooth and rough plane curves. Projectile motion of a particle in a plane. Vector angular velocity. General motion of a rigid body. Central Orbits, Kepler laws of motion. Motion of a particle in three dimensions. Acceleration in terms of different co-ordinate systems.

**Real and Complex Analysis.(03 Questions)** Jacobians, Beta and Gamma functions, Double and Triple integrals, Dirichlet's integrals, change of order of integration in double integrals. Fourier's series: Fourier expansion of piecewise monotonic functions, Properties of Fourier Co-efficients, Dirichlet's conditions, Parseval's identity for Fourier series, Fourier series for even and odd functions, Half range series, Change of Intervals. Extended Complex Plane, Stereographic projection of complex numbers, continuity and differentiability of complex functions, Analytic functions, Cauchy-Riemann equations. Harmonic functions. Mappings by elementary functions: Translation, rotation, Magnification and Inversion. Conformal Mappings, Mobius transformations. Fixed points, Cross ratio, Inverse Points and critical mappings.

**Linear Algebra.(03 Questions)** Vector spaces, subspaces, Sum and Direct sum of subspaces, Linear span, Linearly Independent and dependent subsets of a vector space. Finitely generated vector space, Existence theorem for basis of a finitely generated vector space, Finite dimensional vector spaces, Invariance of the number of elements of bases sets, Dimensions, Quotient space and its dimension. Homomorphism and isomorphism of vector spaces, Linear transformations and linear forms on vector spaces, Vector space of all the linear transformations Dual Spaces, Bidual spaces, annihilator of subspaces of finite dimensional vector spaces, Null Space, Range space of a linear transformation, Rank and Nullity Theorem. Algebra of Linear Transformation, Minimal Polynomial of a linear transformation, Singular and non-singular linear transformations, Matrix of a linear Transformation, Change of basis, Eigen values and

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Eigen vectors of linear transformations. Inner product spaces, Cauchy-Schwarz inequality, Orthogonal vectors, Orthogonal complements, Orthogonal sets and Basis, Bessel's inequality for finite dimensional vector spaces, Gram-Schmidt, Orthogonalization process, Adjoint of a linear transformation and its properties, Unitary linear transformations.

**Numerical Analysis.(03 Questions)** Solution of Algebraic and Transcendental equations: Bisection method, Regula-Falsi method, Secant method, Newton-Raphson's method. Newton's iterative method for finding  $p$ th root of a number, Order of convergence of above methods. Simultaneous linear algebraic equations: Gauss-elimination method, Gauss-Jordan method, Triangularization method (LU decomposition method). Crout's method, Cholesky Decomposition method. Iterative method, Jacobi's method, Gauss-Seidal's method, Relaxation method. Finite Differences operators and their relations. Finding the missing terms and effect of error in a difference tabular values, Interpolation with equal intervals: Newton's forward and Newton's backward interpolation formulae. Interpolation with unequal intervals: Newton's divided difference, Lagrange's Interpolation formulae, Hermite Formula. Central Differences: Gauss forward and Gauss's backward interpolation formulae, Sterling, Bessel Formula. Probability distribution of random variables, Binomial distribution, Poisson's distribution, Normal distribution: Mean, Variance and Fitting. Numerical Differentiation: Derivative of a function using interpolation formulae. Eigen Value Problems: Power method, Jacobi's method, Given's method, House-Holder's method, QR method, Lanczos method. Numerical Integration: Newton-Cote's Quadrature formula, Trapezoidal rule, Simpson's one-third and three-eighths rule, Chebychev formula, Gauss Quadrature formula. Numerical solution of ordinary differential equations: Single step methods-Picard's method. Taylor's series method, Euler's method, Runge-Kutta Methods. Multiple step methods; Predictor-corrector method, Modified Euler's method, Milne-Simpson's method.

